INSTRUMENTATION ENGINEERING

CONTROL SYSTEMS AND PROCESS CONTROL

Comprehensive Theory
with Solved Examples *and* Practice Questions

MADE EASY PUBLICATIONS

www.madeeasypublications.org

MADE EASY Publications Pvt. Ltd.

Corporate Office: 44-A/4, Kalu Sarai (Near Hauz Khas Metro Station), New Delhi-110016 | **Ph. :** 9021300500

Email : infomep@madeeasy.in | **Web :** www.madeeasypublications.org

Control Systems and Process Control

EDITIONS

First Edition : 2015
Second Edition : 2016
Third Edition : 2017
Fourth Edition : 2018
Fifth Edition : 2019
Sixth Edition : 2020
Seventh Edition : 2021
Eighth Edition : 2022
Ninth Edition : 2023
Tenth Edition : 2024
Eleventh Edition : 2025

Twelfth Edition : 2026

CONTENTS

Control Systems and Process Control

Control Systems : Section-A

CHAPTER 7

Time Domain Analysis of Control Systems 109-185

CHAPTER 8

Stability Analysis of Linear Control Systems 186-211

CHAPTER 9

The Root Locus Technique 212-249

CHAPTER 10

Frequency Domain Analysis of Control Systems 250-353

CHAPTER 11

Industrial Controllers and Compensators 354-396

CHAPTER 7

CHAPTER 8

CHAPTER 9

CHAPTER 10

CHAPTER 11

CHAPTER 12

CHAPTER 13

CHAPTER

1

Introduction

Section-A

Control System : Control system is that means by which any quantity of interest in a machine, mechanism or other equipment is maintained or altered in accordance with a desired manner.

Control system can also be defined as the combination of elements arranged in a planned manner wherein each element causes an effect to produce a desired output.

Figure shows the general diagrammatic representation of a typical control system. For the automobile driving system, the input (command) signal is the force on the accelerator pedal which through linkages causes the carburettor value to open (close) so as to increase or decrease fuel (liquid form) flow to the engine bring the engine-vehicle speed (controlled variable) to the desired value.

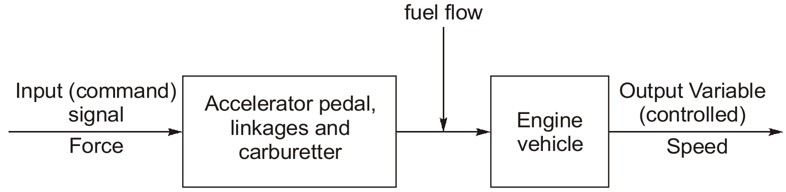


Fig. : *The basic control system*

The diagrammatic representation of above figure is known as block diagram representation wherein each block represents an element, a plant, mechanism, devices etc., whose inner details are not indicated. Each block has an input and output signal which are linked by a relationship characterizing the block. It may be noted that the signal flow through the block is unidirectional.

Control systems are classified into two general categories as Open-loop and closed-loop systems.

1.1 OPEN LOOP CONTROL SYSTEMS

An open loop control system is one in which the control action is independent of the output.

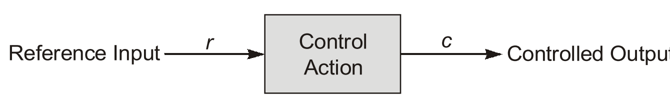


Fig. : *Open-loop control system*

This is the simplest and most economical type of control system and does not have any feedback arrangement.

Some common examples of open-loop control systems are :

(a) Traffic light controller
(b) Electric washing machine
(c) Automatic coffee server
(d) Bread toaster

Advantages of Open Loop Control Systems

(a) It is simple in construction and design.
(b) It costs less time and money to maintain.
(c) These systems are convenient to use when output is difficult to measure.

Disadvantages of Open Loop Control Systems

(a) Inaccurate : With no feedback system, it is very inaccurate.
(b) Unrealiable : It is unreliable as it cannot adapt to uncertainties.
(c) The effect of parameter variation and external noise is more

Note: Open loop control systems does not require performance analysis.

1.2 CLOSED LOOP CONTROL SYSTEMS

A *closed loop control system* is one in which the control action is somehow dependent on the output.

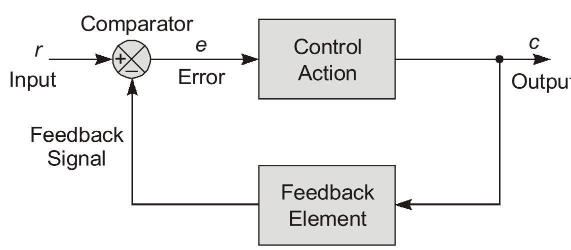


***Fig. :** Closed loop control system*

The closed loop system has same basic features as of open loop system with an additional feedback feature. The actual output is measured and a signal corresponding to this measurement is fedback to the input section, where it is with the input to obtain the desired output.

Some common examples of closed loop control systems are:

(a) Electric iron
(b) DC motor speed control
(c) A missile launching system (direction of missile changes with the location of moving target)
(d) Radar tracking system
(e) Human respiratory system
(f) Autopilot system
(g) Economic inflation

Advantages of Closed Loop Control Systems

(a) Accurate and reliable

(b) Reduced effect of parameter variation

(c) Bandwidth of the system can be increased with negative feedback

(d) Reduced effect of non-linearities

Disadvantages of Closed Loop Control Systems

(a) The system is complex and costly

(b) System may become unstable

(c) Gain of the system reduces with negative feedback

REMEMBER

- Feedback is not used for improving stability
- An open loop stable system may also become unstable when negative feedback is applied
- Except oscillators, in positive feedback, we have always unstable systems.

1.3 DIFFERENCE BETWEEN PERFORMANCE OF OPEN LOOP CONTROL SYSTEM AND CLOSED CONTROL SYSTEMS

Open Loop Control System	Closed Loop Control System
1. Open loop system is not accurate because behaviour of the system does not change if its output changes.	1. Closed loop system is accurate because behaviour of the system does change if its output changes.
2. It either has no sense or it has partial sense but not complete sense.	2. It has complete sense.
3. Time constant of the open loop control system is larger due to which transients takes large time to die out hence open loop system is slow.	3. Time constant of the closed loop control system is smaller due to which transient dies out rapidly. Hence, closed loop control system is fast.
4. Effect of external disturbance and internal parameter variations is more in open loop system, i.e., open loop is more sensitive.	4. Effect of external disturbance and internal parameter variations is less in closed loop system, i.e., closed loop system is less sensitive.
5. Open loop system are simple and economical.	5. Closed loop system is complex and expensive.
6. Open loop system is stable, can become unstable but cannot be stabilized.	6. Closed loop system is stable, can become unstable but can be stabilized.

EXAMPLE : 1.1

Match List-I (Physical action or activity) with List-II (Category of system) and select the correct code:

List - I :

A. Human respiration system

B. Pointing of an object with a finger

C. A man driving a car

D. A thermostatically controlled room heater

List - II :

1. Man-made control system
2. Natural including biological control system
3. Control system whose components are both man-made and natural

Codes:

	A	B	C	D
(a)	2	2	3	1
(b)	3	1	2	1
(c)	3	2	2	3
(d)	2	1	3	3

Solution : (a)

1.4 LAPLACE TRANSFORMATION

S.No.	$f(t)$	$F(s) = L[f(t)]$
1.	$\delta(t)$ unit impulse at $t = 0$	1
2.	$u(t)$ unit step at $t = 0$	$\frac{1}{s}$
3.	$u(t-T)$ unit step at $t = T$	$\frac{1}{s}e^{-sT}$
4.	t	$\frac{1}{s^2}$
5.	$\frac{t^2}{2}$	$\frac{1}{s^3}$
6.	t^n	$\frac{n!}{s^{n+1}}$
7.	e^{at}	$\frac{1}{s-a}$
8.	e^{-at}	$\frac{1}{s+a}$
9.	$t\,e^{at}$	$\frac{1}{(s-a)^2}$
10.	$t\,e^{-at}$	$\frac{1}{(s+a)^2}$
11.	$t^n\,e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
12.	$\sin\omega t$	$\frac{\omega}{s^2+\omega^2}$
13.	$\cos\omega t$	$\frac{s}{s^2+\omega^2}$

Table : *Table of Laplace Transform Pairs*

In order to transform a given function of time $f(t)$ into its corresponding Laplace transform first multiply $f(t)$ by e^{-st}, s being a complex number ($s = \sigma + j\omega$). Integrate this product with respect to time with limits from zero to ∞. This integration results in Laplace transform of $f(t)$, which is denoted by $F(s)$ or $\mathcal{L} f[(t)]$.

The mathematical expression for Laplace transform is,

$$\mathcal{L} f[(t)] = F(s),\ t \geq 0$$

where, $$F(s) = \int_0^{\infty} f(t).e^{-st} dt$$

The original time function $f(t)$ is obtained back from the Laplace transform by a process called inverse Laplace transformation and denoted as $\mathcal{L}^{-1}$

Thus, $$\mathcal{L}^{-1}[\mathcal{L} f(t)] = \mathcal{L}^{-1}[F(s)] = f(t)$$

The time function $f(t)$ and its Laplace transform $F(s)$ form a transform pair.

Basic Laplace Transform Theorems

Basic theorems of Laplace transform are given below:

(a) Laplace Transform of Linear Combination:

$$\mathcal{L}[af_1(t) + bf_2(t)] = aF_1(s) + bF_2(s)$$

where $f_1(t)$, $f_2(t)$ are functions of time and a, b are constants.

(b) If the Laplace Transform of $f(t)$ is $F(s)$, then:

(*i*) $$\mathcal{L}\left[\frac{df(t)}{dt}\right] = [sF(s) - f(0^+)]$$

(*ii*) $$\mathcal{L}\left[\frac{d^2f(t)}{dt^2}\right] = [s^2F(s) - sf(0^+) - f'(0^+)]$$

(*iii*) $$\mathcal{L}\left[\frac{d^3f(t)}{dt^3}\right] = [s^3F(s) - s^2 f(0^+) - sf'(0^+) - f''(0^+)]$$

where $f(0^+)$, $f'(0^+)$, $f''(0^+)$ ... are the values of $f(t)$, $\frac{df(t)}{dt}$, $\frac{d^2f(t)}{dt^2}$ ... at $t = (0^+)$.

(c) If the Laplace Transform of $f(t)$ is $F(s)$, then:

(*i*) $$\mathcal{L}\left[\int f(t)\right] = \left[\frac{F(s)}{s} + \frac{f^{-1}(0^+)}{s}\right]$$

(*ii*) $$\mathcal{L}\left[\iint f(t)\right] = \left[\frac{F(s)}{s^2} + \frac{f^{-1}(0^+)}{s^2} + \frac{f^{-2}(0^+)}{s}\right]$$

(*iii*) $$\mathcal{L}\left[\iiint f(t)\right] = \left[\frac{F(s)}{s^3} + \frac{f^{-1}(0^+)}{s^3} + \frac{f^{-2}(0^+)}{s^2} + \frac{f^{-3}(0^+)}{s}\right]$$

where $f^{-1}(0^+)$, $f^{-2}(0^+)$, $f^{-3}(0^+)$ ... are the values of $\int f(t)$, $\iint f(t)$, $\iiint f(t)$ ... at $t = (0^+)$.

(d) If the Laplace Transform of $f(t)$ is $F(s)$, then:

$$\mathcal{L}[e^{\pm at} f(t)] = F(s \mp a)$$

(e) If the Laplace Transform of $f(t)$ is $F(s)$, then:

$$\mathcal{L}[t\,f(t)] = -\frac{d}{ds}F(s)$$

(f) Initial Value Theorem:

$$\lim_{t \to 0} f(t) = \lim_{s \to \infty} s\mathcal{L}[f(t)]$$

or

$$\lim_{t \to 0} f(t) = \lim_{s \to \infty} sF(s)$$

(*g*) Final Value Theorem:

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} s\mathcal{L}[f(t)]$$

or

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s)$$

NOTE: The final value theorem gives the final value ($t \to \infty$) of a time function using its Laplace transform and as such very useful in the analysis of control systems. However, if the denominator of $sF(s)$ has any root having real part as zero or positive, then the final value theorem is not valid.

EXAMPLE : 1.2

Laplace transform of $\sin(\omega t + \alpha)$ is

(a) $\dfrac{s\cos\alpha + \omega \sin\alpha}{s^2 + \omega^2}$

(b) $\dfrac{\omega}{s^2 + \omega^2}\cos\alpha$

(c) $\dfrac{s}{s^2 + \omega^2}\sin\alpha$

(d) $\dfrac{s\sin\alpha + \omega\cos\alpha}{s^2 + \omega^2}$

Solution : (d)

$$\sin(\omega t + \alpha) = \sin\omega t\cos\alpha + \cos\omega t\sin\alpha$$

$$\mathcal{L}\{\sin(\omega t + \alpha)\} = \frac{\omega\cos\alpha}{s^2 + \omega^2} + \frac{s\sin\alpha}{s^2 + \omega^2}$$

$$= \frac{s\sin\alpha + \omega\cos\alpha}{s^2 + \omega^2}$$

EXAMPLE : 1.3

Given $L[f(t)] = F(s) = \int_0^\infty f(t)e^{-st}dt$, which of the following expressions are correct?

1. $\mathcal{L}\{f(t-a)u(t-a)\} = F(s)e^{-sa}$
2. $\mathcal{L}\{tf(t)\} = \dfrac{-dF(s)}{ds}$
3. $\mathcal{L}\{(t-a)f(t)\} = as\,F(s)\}$
4. $\mathcal{L}\left\{\dfrac{df(t)}{dt}\right\} = sF(s) - f(0^+)$

Select the correct answer using the codes given below

(a) 1, 2 and 3
(b) 1, 2 and 4
(c) 2, 3 and 4
(d) 1, 3 and 4

Solution : (b)

These are the properties of Laplace transform.

EXAMPLE : 1.4

Match List-I [Function in time domain $f(t)$] with List -II [Property] and select the correct answer using the code given below the lists:

List-I	List-II
A. $\sin \omega_0 t\, u(t-t_0)$	1. $\dfrac{\omega_0}{s^2+\omega_0^2}$
B. $\sin \omega_0(t-t_0)\, u(t-t_0)$	2. $\left\{\dfrac{\omega_0}{s^2+\omega_0^2}\right\} e^{-t_0 s}$
C. $\sin \omega_0 (t-t_0)\, u(t)$	3. $\dfrac{e^{-t_0 s}}{\sqrt{s^2+\omega_0^2}} \sin\left(\omega_0 t_0 + \tan^{-1}\dfrac{\omega_0}{s}\right)$
D. $\sin \omega_0 t\, u(t)$	4. $-\dfrac{1}{\sqrt{s^2+\omega_0^2}} \sin\left(\omega_0 t_0 - \tan^{-1}\dfrac{\omega_0}{s}\right)$

Codes:

	A	B	C	D
(a)	3	1	4	2
(b)	4	2	3	1
(c)	3	2	4	1
(d)	4	1	3	2

Solution : (c)

EXAMPLE : 1.5

$F(s) = \dfrac{(1-e^{-sT})}{s}$ is the Laplace transform of

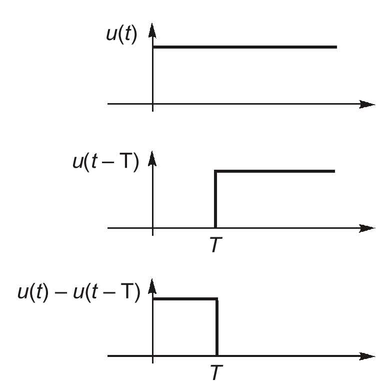
(a) a pulse of width T
(b) a square wave of period T
(c) a unit step delayed by T
(d) a ramp delayed by T

Solution : (a)

$$F(s) = \frac{1-e^{-sT}}{s}$$

$$= \frac{1}{s} - e^{-sT}\left(\frac{1}{s}\right)$$

$$\therefore \quad f(t) = u(t) - u(t-T)$$



■■■■