

# INSTRUMENTATION ENGINEERING

## CONTROL SYSTEMS AND PROCESS CONTROL



Comprehensive Theory  
*with Solved Examples and Practice Questions*





**MADE EASY Publications Pvt. Ltd.**

**Corporate Office:** 44-A/4, Kalu Sarai (Near Hauz Khas Metro Station), New Delhi-110016 | **Ph. :** 9021300500

**Email :** infomep@madeeasy.in | **Web :** www.madeeasypublications.org

## Control Systems and Process Control

Copyright © by MADE EASY Publications Pvt. Ltd.

All rights are reserved. No part of this publication may be reproduced, stored in or introduced into a retrieval system, or transmitted in any form or by any means (electronic, mechanical, photo-copying, recording or otherwise), without the prior written permission of the above mentioned publisher of this book.



**MADE EASY Publications Pvt. Ltd.** has taken due care in collecting the data and providing the solutions, before publishing this book. In spite of this, if any inaccuracy or printing error occurs then **MADE EASY Publications Pvt. Ltd.** owes no responsibility. We will be grateful if you could point out any such error. Your suggestions will be appreciated.

### EDITIONS

First Edition : 2015  
Second Edition : 2016  
Third Edition : 2017  
Fourth Edition : 2018  
Fifth Edition : 2019  
Sixth Edition : 2020  
Seventh Edition : 2021  
Eighth Edition : 2022  
Ninth Edition : 2023  
Tenth Edition : 2024  
Eleventh Edition : 2025  
**Twelfth Edition : 2026**

# CONTENTS

## Control Systems and Process Control

### Control Systems : Section-A

#### CHAPTER 1

##### Introduction..... 1-7

- 1.1 Open Loop Control Systems ..... 1
- 1.2 Closed Loop Control Systems..... 2
- 1.3 Difference between Performance of Open Loop Control System and Closed Control Systems..... 3
- 1.4 Laplace Transformation ..... 4

#### CHAPTER 2

##### Transfer Function ..... 8-27

- 2.1 Transfer Function and Impulse Response Function..... 8
- 2.2 Standard Test Signals..... 10
- 2.3 Poles and Zeros of a Transfer Function ..... 11
- 2.4 Properties Of Transfer Function..... 13
- 2.5 Methods of Analysis..... 14
- 2.6 DC Gain for Open Loop..... 15
- 2.7 Interacting & Non-Interacting Systems ..... 19
  - Objective Brain Teasers* ..... 21
  - Conventional Brain Teasers* ..... 26

#### CHAPTER 3

##### Block Diagrams ..... 28-48

- 3.1 Block Diagrams : Fundamentals ..... 28
- 3.2 Block Diagram of a Closed-Loop System..... 29
- 3.3 Block Diagram Transformation Theorems..... 31
  - Objective Brain Teasers* ..... 41
  - Conventional Brain Teasers* ..... 44

#### CHAPTER 4

##### Signal Flow Graphs ..... 49-69

- 4.1 Introduction..... 49
- 4.2 Terminology of SFG..... 49
- 4.3 Construction of Signal Flow Graphs..... 51
- 4.4 Mason's Gain Formula ..... 54
  - Objective Brain Teasers* ..... 58
  - Conventional Brain Teasers* ..... 66

#### CHAPTER 5

##### Feedback Characteristics..... 70-83

- 5.1 Feedback and Non-Feedback Systems ..... 70
- 5.2 Effect of Feedback on Overall Gain ..... 71
- 5.3 Effect of Feedback on Sensitivity ..... 72
- 5.4 Effect of Feedback on Stability..... 75
- 5.5 Control Over System Dynamics by the Use of Feedback ..... 76
- 5.6 Control on the Effects of the Disturbance Signals by the Use of Feedback ..... 77
- 5.7 Effect of Noise (Disturbance) Signals..... 78
  - Objective Brain Teasers* ..... 80
  - Conventional Brain Teasers* ..... 83

#### CHAPTER 6

##### Modelling of Control Systems..... 84-108

- 6.1 Mechanical Systems..... 84
- 6.2 Electrical Systems ..... 86
- 6.3 Analogous Systems..... 86
- 6.4 Nodal Method for Writing Differential Equation of Complex Mechanical System ..... 87

6.5	Gear Train.....	87
6.6	Servomechanism .....	89
	<i>Objective Brain Teasers</i> .....	102
	<i>Conventional Brain Teasers</i> .....	105

## CHAPTER 7

### Time Domain Analysis of Control Systems ..... 109-185

7.1	Introduction.....	109
7.2	Transient and Steady State Response .....	109
7.3	Steady State Error .....	111
7.4	Static Error Coefficients .....	112
7.5	Dynamic (or Generalised) Error Coefficients.....	120
7.6	Relationship between Static and Dynamic Error Constants .....	121
7.7	Transients State Analysis .....	123
7.8	Dominant Poles of Transfer Functions .....	144
	<i>Objective Brain Teasers</i> .....	152
	<i>Conventional Brain Teasers</i> .....	166

## CHAPTER 8

### Stability Analysis of Linear Control Systems..... 186-211

8.1	The Concept of Stability .....	186
	<i>Objective Brain Teasers</i> .....	204
	<i>Conventional Brain Teasers</i> .....	208

## CHAPTER 9

### The Root Locus Technique ..... 212-249

9.1	Introduction.....	212
9.2	Angle and Magnitude Conditions .....	213
9.3	Construction Rules of Root Locus .....	214
9.4	Gain Margin and Phase Margin from Root Locus Plot.....	222
9.5	Effects of Adding Poles and Zeros to $G(s)H(s)$ .....	225
9.6	Complementary Root Locus (CRL) or Inverse Root Locus (IRL).....	226
	<i>Objective Brain Teasers</i> .....	229
	<i>Conventional Brain Teasers</i> .....	237

## CHAPTER 10

### Frequency Domain Analysis of Control Systems ..... 250-353

10.1	Introduction.....	250
10.2	Advantages of Frequency Response .....	250
10.3	Frequency Response Analysis of Second Order Control System.....	251
10.4	Frequency-Domain Specifications.....	253
10.5	Correlation between Step Response and Frequency Response in the Standard Order System.....	255
10.6	Frequency Domain Analysis of Dead Time or Transportation Lag Elements .....	258
10.7	Relative Stability: Gain Margin & Phase Margin.....	260
10.8	Gain Margin and Phase Margin for Second Order Control System.....	262
10.9	Graphical Methods of Frequency Domain Analysis.....	268
10.10	Polar Plots.....	268
10.11	Stability from Polar Plots .....	275
10.12	Effect of (Open Loop) Gain on Stability .....	276
10.13	Gain Phase Plot.....	277
10.14	Theory of Nyquist Criterion.....	279
10.15	Bode Plots.....	295
10.16	Basic Factors of $G(j\omega)H(j\omega)$ .....	295
10.17	General Procedure for Constructing the Bode Plots.....	300
	<i>Objective Brain Teasers</i> .....	308
	<i>Conventional Brain Teasers</i> .....	327

## CHAPTER 11

### Industrial Controllers and Compensators ..... 354-396

11.1	Introduction to Compensators .....	354
11.2	Lead Compensator .....	357
11.3	Lag Compensator .....	359
11.4	Comparison of Lead and Lag Compensators .....	361
11.5	Lag-Lead Compensator .....	361
11.6	Design by Gain Adjustment.....	369
11.7	Industrial Controllers.....	372
11.8	Proportional ( $P$ ) Controller .....	373
11.9	Integral ( $I$ ) Controller (Reset Mode).....	374
11.10	Derivative ( $D$ ) Controller (Rate Mode).....	375

11.11	Proportional Integral (P-I) Controller.....	377
11.12	Proportional Derivative (P-D) Controller .....	379
11.13	Proportional Integral Derivative (P-I-D) Controller .....	380
11.14	Op-Amp based Realisation of Controllers .....	381
	<i>Objective Brain Teasers</i> .....	387
	<i>Conventional Brain Teasers</i> .....	393

## CHAPTER 12

### State Variable Analysis.....397-450

12.1	Introduction.....	397
12.2	State Space Representation of Control System .....	397
12.3	Special Case: State Equation for Case that Involves Derivative of Input .....	399
12.4	State-Space Representation using Physical Variables - Physical Variable Model .....	399
12.5	Procedure for Deriving State Model for a Given Physical System .....	403
12.6	State Model from Transfer Function.....	403
12.7	State Model from Signal Flow Graph.....	416
12.8	Transfer Function from State Model .....	418
12.9	Stability from State Model.....	420
12.10	Solution of State Equations.....	421
12.11	Properties of State Transition Matrix [ $\phi(t) = e^{At}$ ] .....	422
12.12	Cayley-Hamilton Theorem.....	427
12.13	Controllability and Observability .....	428
12.14	State Variable Feedback .....	429
	<i>Objective Brain Teasers</i> .....	433
	<i>Conventional Brain Teasers</i> .....	440

## Process Control : Section-B

### CHAPTER 1

#### Introduction..... 451-454

1.1	Process.....	451
1.2	Automatic Control .....	452
1.3	Feed Forward Control Technique.....	453
	<i>Objective Brain Teasers</i> .....	454

### CHAPTER 2

#### Modeling of a Chemical Process..... 455-461

2.1	Need of a Mathematical Modelling.....	455
2.2	State Variables and State Equations for Chemical Process.....	455
2.3	Continuous Stirred Tank Reactor (CSTR).....	456
	<i>Objective Brain Teasers</i> .....	461

### CHAPTER 3

#### Dynamic Behaviour of First and Second order Systems.....462-471

3.1	First Order System .....	462
3.2	Second Order System.....	465
3.3	Multicapacity Process (as second order systems) .....	467
	<i>Objective Brain Teasers</i> .....	471

### CHAPTER 4

#### Dead Time in a Process..... 472-477

	<i>Objective Brain Teasers</i> .....	477
--	--------------------------------------	-----

### CHAPTER 5

#### Feedback Control.....478-489

5.1	Introduction.....	478
5.2	Types of Feedback Controllers .....	479
5.3	Composite Control Actions .....	487
	<i>Objective Brain Teasers</i> .....	489

### CHAPTER 6

#### Different Control Actions..... 490-501

6.1	Cascade Control .....	490
6.2	Selective Control System .....	497
6.3	Split-Range Control.....	499
	<i>Objective Brain Teasers</i> .....	501

**CHAPTER 7****Feedforward Control..... 502-505**

- 7.1 Ratio Control..... 504
- Objective Brain Teasers*..... 505

**CHAPTER 8****Designing of Feedback Controllers..... 506-511**

- 8.1 Introduction..... 506
- 8.2 Different Performance Criterion..... 506
- 8.3 Controller Tuning ..... 509
- Objective Brain Teasers*..... 511

**CHAPTER 9****Controlling Elements.....512-517**

- 9.1 Introduction..... 512
- 9.2 Fluid Flow Through Control Valves..... 513
- 9.3 Types of Stem-Control Valve..... 513
- Objective Brain Teasers*..... 517

**CHAPTER 10****Tuning of PID Controllers ..... 518-521**

- 10.1 Introduction ..... 518
- 10.2 Reaction Curve Technique ..... 519
- 10.3 Closed Loop Technique  
(Continuous Cycling Method) ..... 519
- 10.4 Closed Loop Technique  
(Damped oscillation method) ..... 520
- 10.5 General Comments about Controller Tuning ..... 521
- 10.6 Integration windup and Bumpless transfer ..... 521

**CHAPTER 11****Basics of Data Acquisition Systems .... 522-525**

- 11.1 Data Acquisition System ..... 522
- 11.2 Telemetry..... 524
- Objective Brain Teasers*..... 525

**CHAPTER 12****Programmable Logic Controllers .....526-575**

- 12.1 Introduction..... 526
- 12.2 Parts of a PLC ..... 527
- 12.3 Principles of Operation..... 530
- 12.4 Modifying the Operation ..... 531
- 12.5 PLCs versus Computers..... 531
- 12.6 PLC Size and Application..... 532
- 12.7 Logic Programming ..... 533
- 12.8 Varieties of PLC ..... 537
- 12.9 Ladder Diagram (LD) Programming..... 538
- 12.10 Contacts and Coils..... 538
- 12.11 Types of Logic Gates using PLC Ladder ..... 540
- 12.12 Diagram PLC Programming ..... 552
- 12.13 Counter..... 554
- 12.14 Timers..... 556
- 12.15 Data Comparison Instructions ..... 558
- 12.16 Math Instructions..... 560
- 12.17 Human-Machine Interfaces..... 563
- 12.18 Scan Cycle of PLC ..... 566
- Objective Brain Teasers*..... 568

**CHAPTER 13****Distributed Control Systems (DCS) .....576-586**

- 13.1 Introduction..... 576
- 13.2 Comparison between DCS and PLC..... 577
- 13.3 PLC vs. DCS: Which is Right for Your Operation? .... 578
- 13.4 Automation Tools..... 579
- 13.5 SCADA..... 580
- 13.6 PLC vs. SCADA ..... 582
- 13.7 Remote Terminal Units ..... 582
- 13.8 Smart Instrument..... 584
- Objective Brain Teasers*..... 585

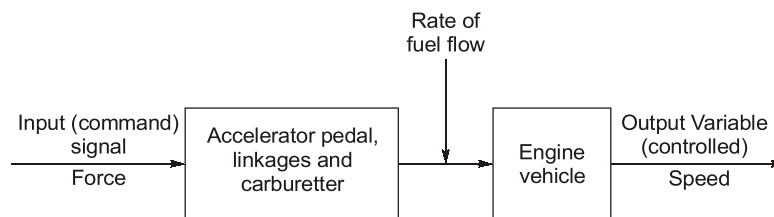
# Introduction

## Section-A

**Control System :** Control system is that means by which any quantity of interest in a machine, mechanism or other equipment is maintained or altered in accordance with a desired manner.

Control system can also be defined as the combination of elements arranged in a planned manner wherein each element causes an effect to produce a desired output.

Figure shows the general diagrammatic representation of a typical control system. For the automobile driving system, the input (command) signal is the force on the accelerator pedal which through linkages causes the carburettor valve to open (close) so as to increase or decrease fuel (liquid form) flow to the engine bring the engine-vehicle speed (controlled variable) to the desired value.



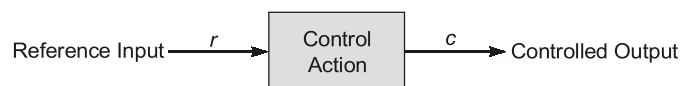
*Fig. : The basic control system*

The diagrammatic representation of above figure is known as block diagram representation wherein each block represents an element, a plant, mechanism, devices etc., whose inner details are not indicated. Each block has an input and output signal which are linked by a relationship characterizing the block. It may be noted that the signal flow through the block is unidirectional.

Control systems are classified into two general categories as Open-loop and closed-loop systems.

## 1.1 OPEN LOOP CONTROL SYSTEMS

*An open loop control system* is one in which the control action is independent of the output.



*Fig. : Open-loop control system*

This is the simplest and most economical type of control system and does not have any feedback arrangement.

Some common examples of open-loop control systems are :

- Traffic light controller
- Electric washing machine
- Automatic coffee server
- Bread toaster

#### Advantages of Open Loop Control Systems

- It is simple in construction and design.
- It costs less time and money to maintain.
- These systems are convenient to use when output is difficult to measure.

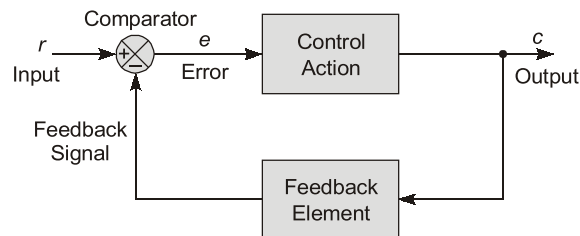
#### Disadvantages of Open Loop Control Systems

- Inaccurate : With no feedback system, it is very inaccurate.
- Unreliable : It is unreliable as it cannot adapt to uncertainties.
- The effect of parameter variation and external noise is more

**Note:** Open loop control systems does not require performance analysis.

## 1.2 CLOSED LOOP CONTROL SYSTEMS

A closed loop control system is one in which the control action is somehow dependent on the output.



**Fig. :** Closed loop control system

The closed loop system has same basic features as of open loop system with an additional feedback feature. The actual output is measured and a signal corresponding to this measurement is fed back to the input section, where it is with the input to obtain the desired output.

Some common examples of closed loop control systems are:

- Electric iron
- DC motor speed control
- A missile launching system (direction of missile changes with the location of moving target)
- Radar tracking system
- Human respiratory system
- Autopilot system
- Economic inflation

#### Advantages of Closed Loop Control Systems

- Accurate and reliable

- (b) Reduced effect of parameter variation
- (c) Bandwidth of the system can be increased with negative feedback
- (d) Reduced effect of non-linearities

**Disadvantages of Closed Loop Control Systems**

- (a) The system is complex and costly
- (b) System may become unstable
- (c) Gain of the system reduces with negative feedback



**REMEMBER**

- Feedback is not used for improving stability
- An open loop stable system may also become unstable when negative feedback is applied
- Except oscillators, in positive feedback, we have always unstable systems.

**1.3 DIFFERENCE BETWEEN PERFORMANCE OF OPEN LOOP CONTROL SYSTEM AND CLOSED CONTROL SYSTEMS**

Open Loop Control System	Closed Loop Control System
1. Open loop system is not accurate because behaviour of the system does not change if its output changes.	1. Closed loop system is accurate because behaviour of the system does change if its output changes.
2. It either has no sense or it has partial sense but not complete sense.	2. It has complete sense.
3. Time constant of the open loop control system is larger due to which transients takes large time to die out hence open loop system is slow.	3. Time constant of the closed loop control system is smaller due to which transient dies out rapidly. Hence, closed loop control system is fast.
4. Effect of external disturbance and internal parameter variations is more in open loop system, i.e., open loop is more sensitive.	4. Effect of external disturbance and internal parameter variations is less in closed loop system, i.e., closed loop system is less sensitive.
5. Open loop system are simple and economical.	5. Closed loop system is complex and expensive.
6. Open loop system is stable, can become unstable but cannot be stabilized.	6. Closed loop system is stable, can become unstable but can be stabilized.

**EXAMPLE : 1.1**

Match List-I (Physical action or activity) with List-II (Category of system) and select the correct code:

List - I :

- A. Human respiration system
- B. Pointing of an object with a finger
- C. A man driving a car
- D. A thermostatically controlled room heater

List - II :

- 1. Man-made control system
- 2. Natural including biological control system
- 3. Control system whose components are both man-made and natural

Codes:

	A	B	C	D
(a)	2	2	3	1
(b)	3	1	2	1
(c)	3	2	2	3
(d)	2	1	3	3

Solution : (a)

## 1.4 LAPLACE TRANSFORMATION

S.No.	$f(t)$	$F(s) = L[f(t)]$
1.	$\delta(t)$ unit impulse at $t = 0$	1
2.	$u(t)$ unit step at $t = 0$	$\frac{1}{s}$
3.	$u(t - T)$ unit step at $t = T$	$\frac{1}{s} e^{-sT}$
4.	$t$	$\frac{1}{s^2}$
5.	$\frac{t^2}{2}$	$\frac{1}{s^3}$
6.	$t^n$	$\frac{n!}{s^{n+1}}$
7.	$e^{at}$	$\frac{1}{s - a}$
8.	$e^{-at}$	$\frac{1}{s + a}$
9.	$t e^{at}$	$\frac{1}{(s - a)^2}$
10.	$t e^{-at}$	$\frac{1}{(s + a)^2}$
11.	$t^n e^{-at}$	$\frac{n!}{(s + a)^{n+1}}$
12.	$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$
13.	$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$

**Table :** Table of Laplace Transform Pairs

In order to transform a given function of time  $f(t)$  into its corresponding Laplace transform first multiply  $f(t)$  by  $e^{-st}$ ,  $s$  being a complex number ( $s = \sigma + j\omega$ ). Integrate this product with respect to time with limits from zero to  $\infty$ . This integration results in Laplace transform of  $f(t)$ , which is denoted by  $F(s)$  or  $\mathcal{L}f[(t)]$ .

The mathematical expression for Laplace transform is,

$$\mathcal{L} f(t) = F(s), t \geq 0$$

where,

$$F(s) = \int_0^{\infty} f(t).e^{-st} dt$$

The original time function  $f(t)$  is obtained back from the Laplace transform by a process called inverse Laplace transformation and denoted as  $\mathcal{L}^{-1}$

Thus, 
$$\mathcal{L}^{-1} [\mathcal{L} f(t)] = \mathcal{L}^{-1} [F(s)] = f(t)$$

The time function  $f(t)$  and its Laplace transform  $F(s)$  form a transform pair.

**Basic Laplace Transform Theorems**

Basic theorems of Laplace transform are given below:

**(a) Laplace Transform of Linear Combination:**

$$\mathcal{L}[af_1(t) + bf_2(t)] = aF_1(s) + bF_2(s)$$

where  $f_1(t), f_2(t)$  are functions of time and  $a, b$  are constants.

**(b) If the Laplace Transform of  $f(t)$  is  $F(s)$ , then:**

(i) 
$$\mathcal{L}\left[\frac{df(t)}{dt}\right] = [sF(s) - f(0^+)]$$

(ii) 
$$\mathcal{L}\left[\frac{d^2f(t)}{dt^2}\right] = [s^2F(s) - sf(0^+) - f'(0^+)]$$

(iii) 
$$\mathcal{L}\left[\frac{d^3f(t)}{dt^3}\right] = [s^3F(s) - s^2 f(0^+) - sf'(0^+) - f''(0^+)]$$

where  $f(0^+), f'(0^+), f''(0^+) \dots$  are the values of  $f(t), \frac{df(t)}{dt}, \frac{d^2f(t)}{dt^2} \dots$  at  $t = (0^+)$ .

**(c) If the Laplace Transform of  $f(t)$  is  $F(s)$ , then:**

(i) 
$$\mathcal{L}\left[\int f(t)\right] = \left[\frac{F(s)}{s} + \frac{f^{-1}(0^+)}{s}\right]$$

(ii) 
$$\mathcal{L}\left[\iint f(t)\right] = \left[\frac{F(s)}{s^2} + \frac{f^{-1}(0^+)}{s^2} + \frac{f^{-2}(0^+)}{s}\right]$$

(iii) 
$$\mathcal{L}\left[\iiint f(t)\right] = \left[\frac{F(s)}{s^3} + \frac{f^{-1}(0^+)}{s^3} + \frac{f^{-2}(0^+)}{s^2} + \frac{f^{-3}(0^+)}{s}\right]$$

where  $f^{-1}(0^+), f^{-2}(0^+), f^{-3}(0^+) \dots$  are the values of  $\int f(t), \iint f(t), \iiint f(t) \dots$  at  $t = (0^+)$ .

**(d) If the Laplace Transform of  $f(t)$  is  $F(s)$ , then:**

$$\mathcal{L}[e^{\pm at} f(t)] = F(s \mp a)$$

**(e) If the Laplace Transform of  $f(t)$  is  $F(s)$ , then:**

$$\mathcal{L}[t f(t)] = -\frac{d}{ds} F(s)$$

(f) Initial Value Theorem:

$$\lim_{t \rightarrow 0} f(t) = \lim_{s \rightarrow \infty} s \mathcal{L}[f(t)]$$

or

$$\lim_{t \rightarrow 0} f(t) = \lim_{s \rightarrow \infty} sF(s)$$

(g) Final Value Theorem:

$$\lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} s \mathcal{L}[f(t)]$$

or

$$\lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} sF(s)$$

NOTE



The final value theorem gives the final value ( $t \rightarrow \infty$ ) of a time function using its Laplace transform and as such very useful in the analysis of control systems. However, if the denominator of  $sF(s)$  has any root having real part as zero or positive, then the final value theorem is not valid.

**EXAMPLE : 1.2**Laplace transform of  $\sin(\omega t + \alpha)$  is

(a) 
$$\frac{s \cos \alpha + \omega \sin \alpha}{s^2 + \omega^2}$$

(b) 
$$\frac{\omega}{s^2 + \omega^2} \cos \alpha$$

(c) 
$$\frac{s}{s^2 + \omega^2} \sin \alpha$$

(d) 
$$\frac{s \sin \alpha + \omega \cos \alpha}{s^2 + \omega^2}$$

**Solution : (d)**

$$\begin{aligned} \sin(\omega t + \alpha) &= \sin \omega t \cos \alpha + \cos \omega t \sin \alpha \\ \mathcal{L}\{\sin(\omega t + \alpha)\} &= \frac{\omega \cos \alpha}{s^2 + \omega^2} + \frac{s \sin \alpha}{s^2 + \omega^2} \\ &= \frac{s \sin \alpha + \omega \cos \alpha}{s^2 + \omega^2} \end{aligned}$$

**EXAMPLE : 1.3**Given  $\mathcal{L}[f(t)] = F(s) = \int_0^{\infty} f(t) e^{-st} dt$ , which of the following expressions are correct?

1.  $\mathcal{L}\{f(t-a)u(t-a)\} = F(s)e^{-sa}$

2.  $\mathcal{L}\{tf(t)\} = \frac{-dF(s)}{ds}$

3.  $\mathcal{L}\{(t-a)f(t)\} = as F(s)$

4.  $\mathcal{L}\left\{\frac{df(t)}{dt}\right\} = sF(s) - f(0^+)$

Select the correct answer using the codes given below

(a) 1, 2 and 3

(b) 1, 2 and 4

(c) 2, 3 and 4

(d) 1, 3 and 4

**Solution : (b)**

These are the properties of Laplace transform.

**EXAMPLE : 1.4**

Match List-I [Function in time domain  $f(t)$ ] with List -II [Property] and select the correct answer using the code given below the lists:

- | List-I                             | List-II   |
|------------------------------------|---|
| A. $\sin \omega_0 t u(t-t_0)$      | 1. $\frac{\omega_0}{s^2 + \omega_0^2}$  |
| B. $\sin \omega_0(t-t_0) u(t-t_0)$ | 2. $\left\{ \frac{\omega_0}{s^2 + \omega_0^2} \right\} e^{-t_0 s}$  |
| C. $\sin \omega_0(t-t_0) u(t)$     | 3. $\frac{e^{-t_0 s}}{\sqrt{s^2 + \omega_0^2}} \sin \left( \omega_0 t_0 + \tan^{-1} \frac{\omega_0}{s} \right)$ |
| D. $\sin \omega_0 t u(t)$          | 4. $-\frac{1}{\sqrt{s^2 + \omega_0^2}} \sin \left( \omega_0 t_0 - \tan^{-1} \frac{\omega_0}{s} \right)$         |

Codes:

	A	B	C	D
(a)	3	1	4	2
(b)	4	2	3	1
(c)	3	2	4	1
(d)	4	1	3	2

**Solution : (c)**

**EXAMPLE : 1.5**

$F(s) = \frac{(1 - e^{-sT})}{s}$  is the Laplace transform of

- |                                |                                 |
|--------------------------------|---------------------------------|
| (a) a pulse of width $T$       | (b) a square wave of period $T$ |
| (c) a unit step delayed by $T$ | (d) a ramp delayed by $T$       |

**Solution : (a)**

$$F(s) = \frac{1 - e^{-sT}}{s}$$

$$= \frac{1}{s} - e^{-sT} \left( \frac{1}{s} \right)$$

$\therefore f(t) = u(t) - u(t-T)$

